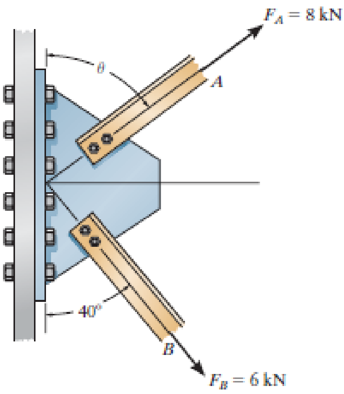
CHAPTER II (SELECTED PROBLEMS)

•2–9. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



Parallelogram Law **:** The parallelogram law of addition is shown in Fig. (a).

Trigonometry **:** Using law of cosines [Fig. (b)], we have

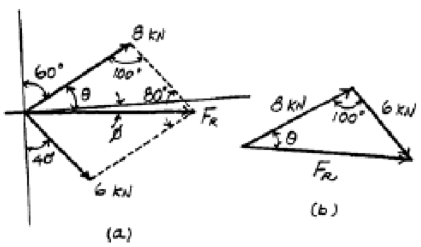
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans}$$

The angle θ can be determined using law of sines [Fig. (b)].

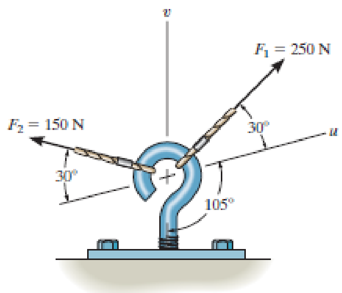
$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16°$$

Thus, the direction ϕ of F_R measured from the x axis is

$$\phi = 33.16° - 30° = 3.16° \qquad \textbf{Ans}$$



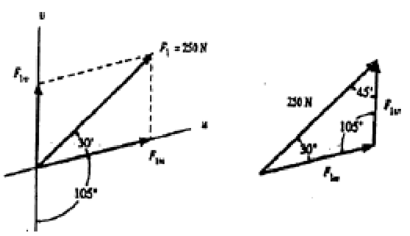
*2–16. Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.



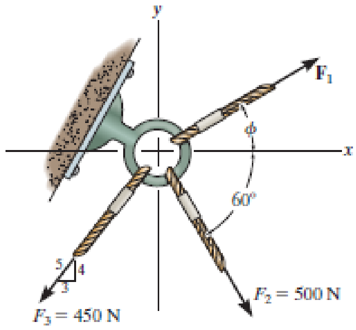
Sine law :

$$\frac{F_{1v}}{\sin 30°} = \frac{250}{\sin 105°} \qquad F_{1v} = 129 \text{ N} \qquad \textbf{Ans}$$

$$\frac{F_{1u}}{\sin 45°} = \frac{250}{\sin 105°} \qquad F_{1u} = 183 \text{ N} \qquad \textbf{Ans}$$



•2–33. If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. a, the x and y components of each force can be written as

$$(F_1)_x = 600\cos 30° = 519.62\text{ N} \quad (F_1)_y = 600\sin 30° = 300\text{ N}$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \quad (F_2)_y = 500\sin 60° = 433.0\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \quad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62\text{ N} \rightarrow$$

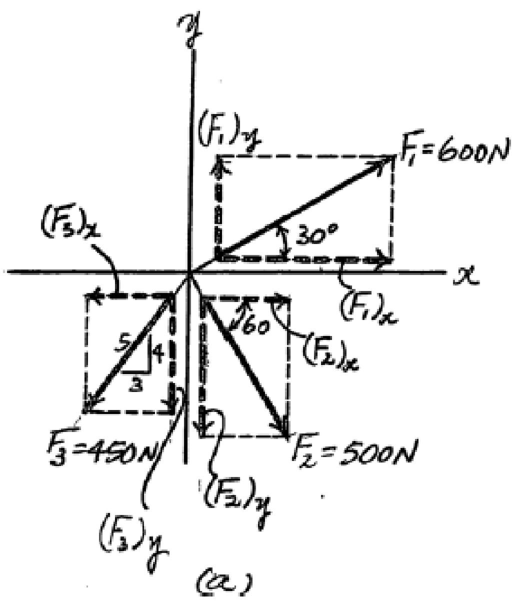
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01\text{N} = 493.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

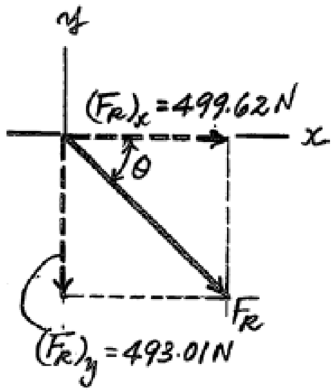
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{499.62^2 + 493.01^2} = 701.91\text{ N} = 702\text{ N}$$ **Ans.**

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6°$$ **Ans.**

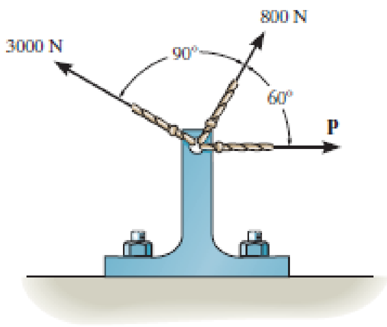


(a)



(b)

2–50. The three forces are applied to the bracket. Determine the range of values for the magnitude of force **P** so that the resultant of the three forces does not exceed 2400 N.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = P + 800\cos 60^\circ - 3000\cos 30^\circ = P - 2198.08$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 800\sin 60^\circ + 3000\sin 30^\circ = 2192.82$

$$F_R = \sqrt{(P - 2198.08)^2 + (2192.82)^2} \le 2400$$

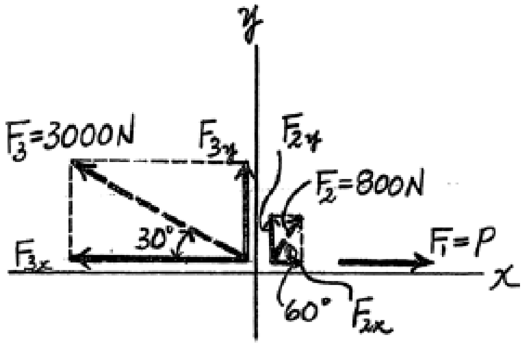
$$(P - 2198.08)^2 + (2192.82)^2 \le (2400)^2$$

$$|(P - 2198.08)| \le 975.47$$

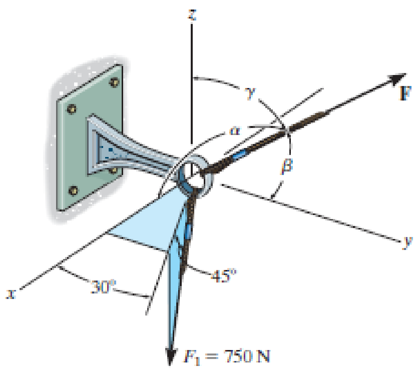
$$-975.47 \le P - 2198.08 \le 975.47$$

$$1222.6\text{ N} \le P \le 3173.5\text{ N}$$

$$1.22\text{ kN} \le P \le 3.17\text{ kN} \qquad \text{Ans}$$



•2–69. If the resultant force acting on the bracket is $\mathbf{F}_R = \{-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Fig. *a*. $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45^\circ\cos 30^\circ(+\mathbf{i}) + 750\cos 45^\circ\sin 30^\circ(+\mathbf{j}) + 750\sin 45^\circ(-\mathbf{k})$$

$$= \{459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}\}\text{N}$$

$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$

$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28 + F\cos\theta_x)\mathbf{i} + (265.17 + F\cos\theta_y)\mathbf{j} + (F\cos\theta_z - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$-300 = 459.28 + F\cos\alpha$$

$$F\cos\alpha = -759.28 \qquad (1)$$

$$650 = 265.17 + F\cos\beta$$

$$F\cos\beta = 384.83 \qquad (2)$$

$$250 = F\cos\gamma - 530.33$$

$$F\cos\gamma = 780.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

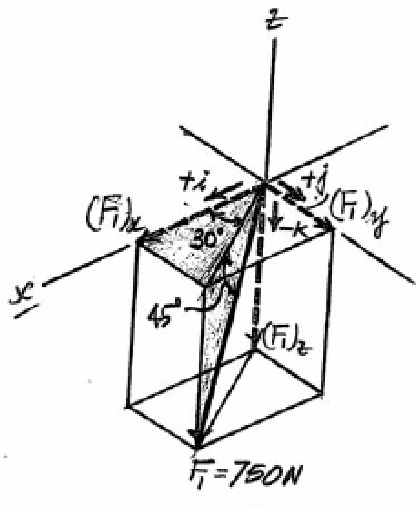
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 1\,333\,518.08 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

$$F = 1154.78\text{ N} = 1.15\text{ kN} \qquad \textbf{Ans.}$$

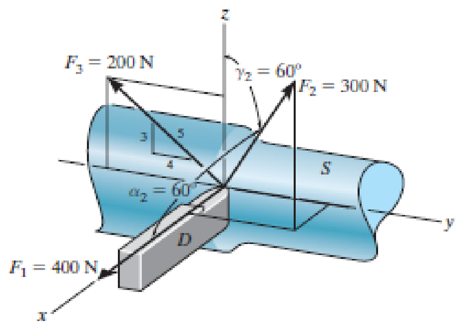
Substituting $F = 1154.78$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 131^\circ \qquad \beta = 70.5^\circ \qquad \gamma = 47.5^\circ \qquad \textbf{Ans.}$$



(a)

•2–73. The shaft S exerts three force components on the die D. Find the magnitude and coordinate direction angles of the resultant force. Force $\mathbf{F}_2$ acts within the octant shown.



$\mathbf{F}_1 = 400\,\mathbf{i}$

Since $\cos^2 60° + \cos^2 \beta_2 + \cos^2 60° = 1$

Solving for the positive root, $\beta_2 = 45°$

$\mathbf{F}_2 = 300\cos 60°\,\mathbf{i} + 300\cos 45°\,\mathbf{j} + 300\cos 60°\,\mathbf{k}$

$= 150\,\mathbf{i} + 212.1\,\mathbf{j} + 150\,\mathbf{k}$

$\mathbf{F}_3 = -200\left(\frac{4}{5}\right)\mathbf{j} + 200\left(\frac{3}{5}\right)\mathbf{k}$

$= -160\,\mathbf{j} + 120\,\mathbf{k}$

Then

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 550\,\mathbf{i} + 52.1\,\mathbf{j} + 270\,\mathbf{k}$

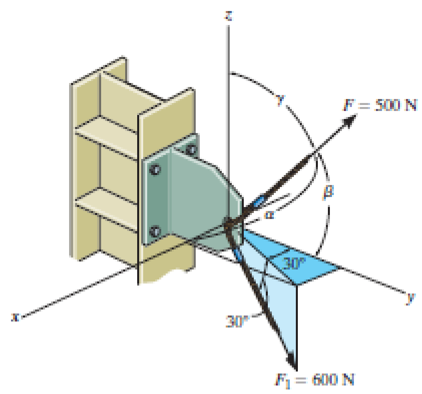
$F_R = \sqrt{(550)^2 + (52.1)^2 + (270)^2} = 614.9\text{ N} = 615\text{ N}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{550}{614.9}\right) = 26.6°$ **Ans**

$\beta = \cos^{-1}\left(\frac{52.1}{614.9}\right) = 85.1°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{270}{614.9}\right) = 64.0°$ **Ans**

2–78. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of $\mathbf{F}$ so that $\beta < 90°$.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$
$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\ \text{N}$$
$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$F_R\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\theta_x\mathbf{i} + 500\cos\theta_y\mathbf{j} + 500\cos\theta_z\mathbf{k})$$
$$F_R\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = 259.81 + 500\cos\alpha$$
$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$
$$F_R = 450 + 500\cos\beta \qquad (1)$$

$$0 = 500\cos\gamma - 300$$
$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

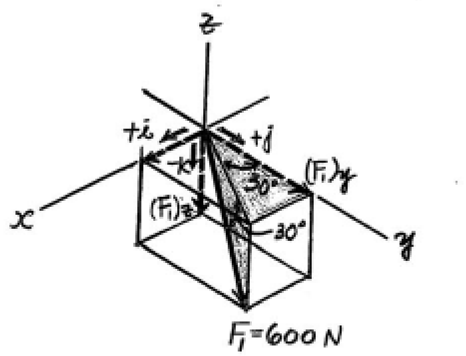
$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

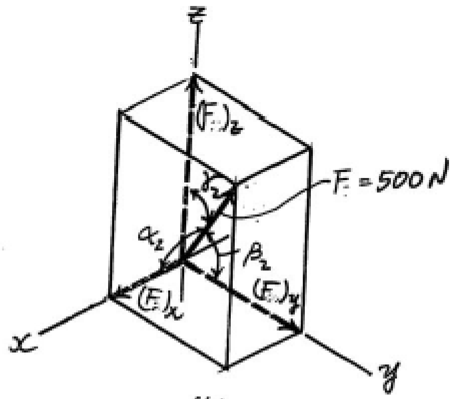
$$F_R = 450 + 500(0.6083) = 754\ \text{N} \qquad \textbf{Ans.}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

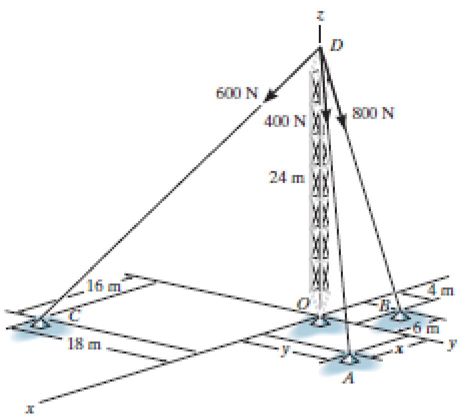


(a)



(b)

*2–96. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\} \text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

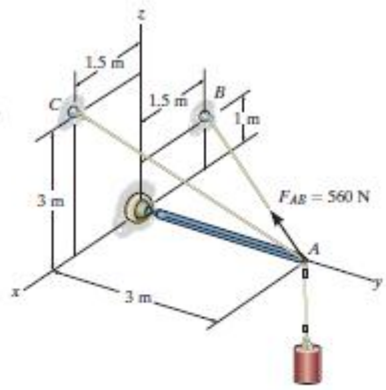
$$= 1501.66 \text{ N} = 1.50 \text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ \qquad \textbf{Ans}$$

*2–112. Determine the projected component of the force $F_{AB} = 560$ N acting along cable AC. Express the result as a Cartesian vector.



Force Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(0-3)^2+(1-0)^2}} = -\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(1.5-0)^2+(0-3)^2+(3-0)^2}} = \frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 560\left(-\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = \{-240\mathbf{i}-480\mathbf{j}+160\mathbf{k}\}\text{N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ is

$$(F_{AB})_{AC} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = (-240\mathbf{i}-480\mathbf{j}+160\mathbf{k})\cdot\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

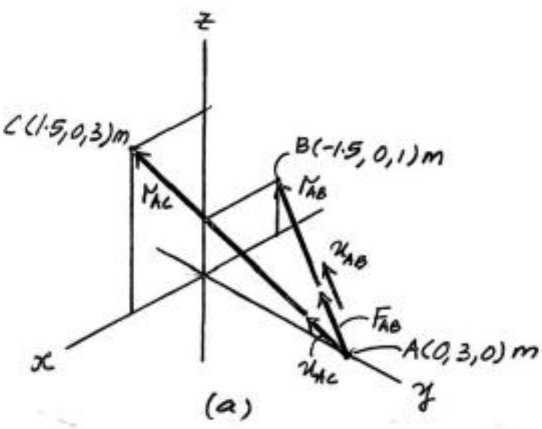
$$= (-240)\left(\frac{1}{3}\right)+(-480)\left(-\frac{2}{3}\right)+160\left(\frac{2}{3}\right)$$

$$= 346.67\text{ N}$$

Thus, $(\mathbf{F}_{AB})_{AC}$ expressed in Cartesian vector form is

$$(\mathbf{F}_{AB})_{AC} = (F_{AB})_{AC}\,\mathbf{u}_{AC} = 346.67\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

$$= \{116\mathbf{i}-231\mathbf{j}+231\mathbf{k}\}\text{N}$$ **Ans.**



(a)