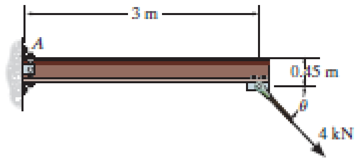
CHAPTER IV (SELECTED PROBLEMS)

4–7. If the moment produced by the 4-kN force about point A is 10 kN · m clockwise, determine the angle θ, where $0° \le \theta \le 90°$.



Resolving the 4 - kN force into its horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -10 = 4\cos\theta(0.45) - 4\sin\theta(3)$$

$$12\sin\theta - 1.8\cos\theta = 10 \qquad (1)$$

Referring to the geometry of Fig. *a*,

$$\cos\phi = \frac{12}{\sqrt{147.24}} \qquad \sin\phi = \frac{1.8}{\sqrt{147.24}} \qquad (2)$$

Dividing Eq. (1) by $\sqrt{147.24}$ yields

$$\frac{12}{\sqrt{147.24}}\sin\theta - \frac{1.8}{\sqrt{147.24}}\cos\theta = \frac{10}{\sqrt{147.24}} \qquad (3)$$

Substituting Eq. (2) into (3) yields

$$\sin\theta\cos\phi - \cos\theta\sin\phi = \frac{10}{\sqrt{147.24}}$$

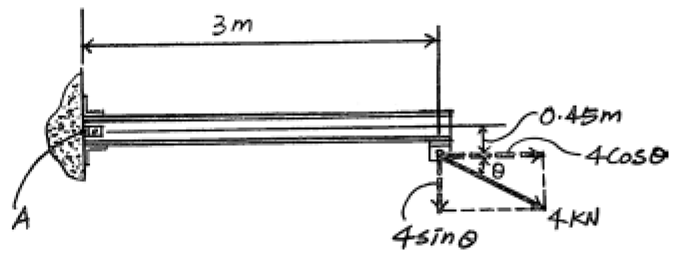
$$\sin(\theta - \phi) = \frac{10}{\sqrt{147.24}}$$

However, $\phi = \tan^{-1}\left(\frac{1.8}{12}\right) = 8.531°$. Thus,

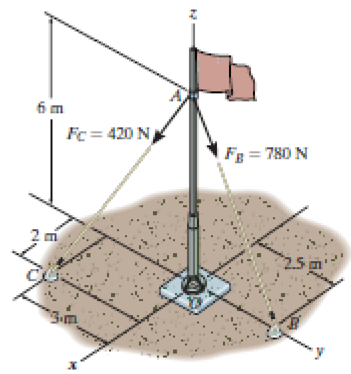
$$\sin(\theta - 8.531°) = \frac{10}{\sqrt{147.24}}$$

$$\theta - 8.531° = 55.50°$$

$$\theta = 64.0° \qquad \textbf{Ans.}$$



***4–40.** Determine the moment produced by force $\mathbf{F}_B$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OB}$ can be used to determine the moment of $\mathbf{F}_B$ about point O.

$\mathbf{r}_{OA} = \{6\mathbf{k}\}$ m $\qquad \mathbf{r}_{OB} = \{2.5\mathbf{j}\}$ m

The force vector $\mathbf{F}_B$ is given by

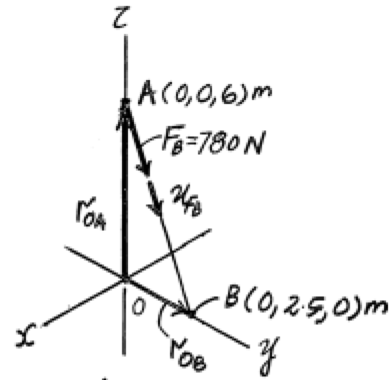
$$\mathbf{F}_B = F_B\mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i}+(2.5-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(2.5-0)^2+(0-6)^2}}\right] = \{300\mathbf{j}-720\mathbf{k}\}\text{N}$$

Vector Cross Product: The moment of $\mathbf{F}_B$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\text{ N}\cdot\text{m} = \{-1.80\mathbf{i}\}\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

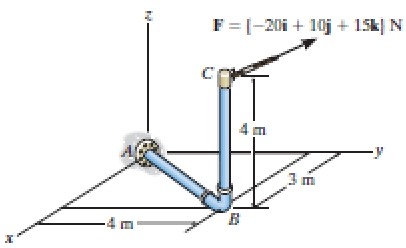
or

$$\mathbf{M}_O = \mathbf{r}_{OB}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 2.5 & 0\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\text{ N}\cdot\text{m} = \{-1.80\mathbf{i}\}\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

***4–56.** Determine the moment produced by force **F** about segment *AB* of the pipe assembly. Express the result as a Cartesian vector.



Moment About Line AB: Either position vector $\mathbf{r}_{AC}$ or $\mathbf{r}_{BC}$ can be conveniently used to determine the moment of **F** about line *AB*.

$\mathbf{r}_{AC} = (3-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k} = [3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}]$ m

$\mathbf{r}_{BC} = (3-3)\mathbf{i} + (4-4)\mathbf{j} + (4-0)\mathbf{k} = [4\mathbf{k}]$ m

The unit vector $\mathbf{u}_{AB}$, Fig. *a*, that specifies the direction of line *AB* is given by

$$\mathbf{u}_{AB} = \frac{(3-0)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (0-0)^2}} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$$

Thus, the magnitude of the moment of **F** about line *AB* is given by

$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 3 & 4 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[4(15) - 10(4)] - \frac{4}{5}[3(15) - (-20)(4)] + 0$$

$$= -88 \text{ N} \cdot \text{m}$$

or

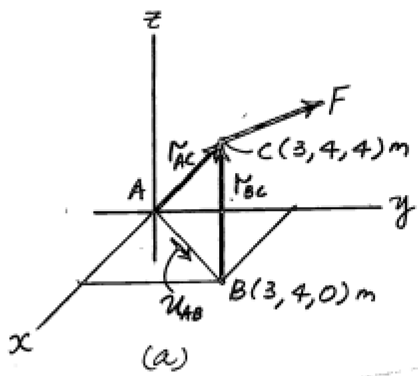
$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[0(15) - 10(4)] - \frac{4}{5}[0(15) - (-20)(4)] + 0$$

$$= -88 \text{ N} \cdot \text{m}$$

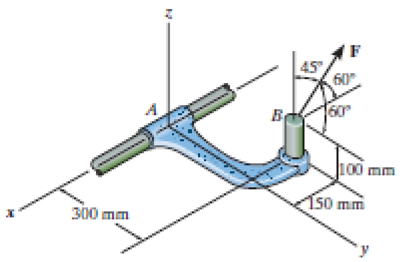
Thus, $\mathbf{M}_{AB}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AB} = M_{AB}\mathbf{u}_{AB} = -88\left(\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}\right) = [-52.8\mathbf{i} - 70.4\mathbf{j}] \text{N} \cdot \text{m}$$ **Ans.**



(a)

4–58. If $F = 450$ N, determine the magnitude of the moment produced by this force about the x axis.



Moment About the x axis: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{CB}$ can be used to determine the moment of **F** about the x axis.

$\mathbf{r}_{AB} = (-0.15-0)\mathbf{i} + (0.3-0)\mathbf{j} + (0.1-0)\mathbf{k} = [-0.15\mathbf{i} + 0.3\mathbf{j} + 0.1\mathbf{k}]$ m

$\mathbf{r}_{CB} = [(-1.5-(-0.15))]\mathbf{i} + (0.3-0)\mathbf{j} + (0.1-0)\mathbf{k} = [0.3\mathbf{j} + 0.1\mathbf{k}]$ m

The force vector **F** is given by

$\mathbf{F} = 450(-\cos 60°\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\mathbf{k}) = [-225\mathbf{i} + 225\mathbf{j} + 318.20\mathbf{k}]$N

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

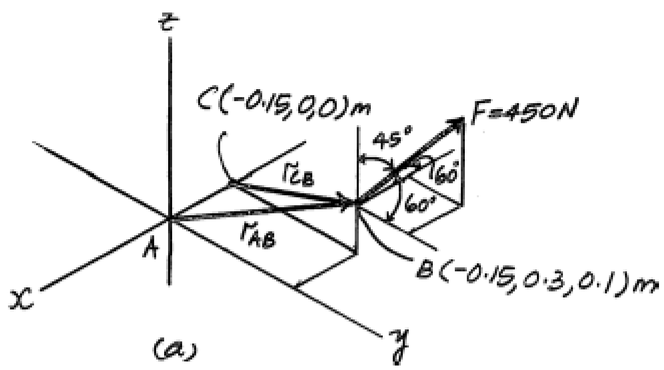
$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20) - (225)(0.1)] + 0 + 0 = 73.0\ \text{N}\cdot\text{m}$$ **Ans.**

or

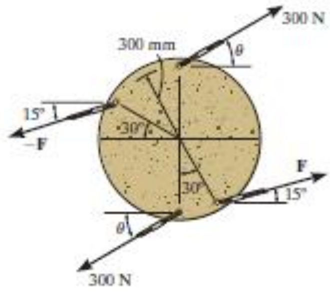
$$M_x = \mathbf{i}\cdot\mathbf{r}_{CB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20) - (225)(0.1)] + 0 + 0 = 73.0\ \text{N}\cdot\text{m}$$ **Ans.**



(a)

4–79. If $F = 200$ N, determine the required angle θ so that the resultant couple moment is zero.

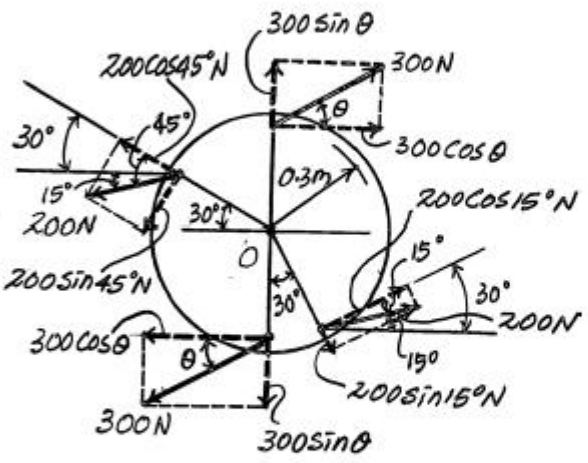


By resolving the 300 - N and 200 -N couples into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point O,

$$\circlearrowleft +(M_c)_R = \Sigma M_O; \quad 0 = 200\sin 45°(0.3) + 200\cos 15°(0.3) - 300\cos\theta(0.3) - 300\cos\theta(0.3)$$

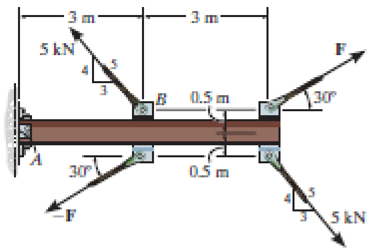
$$\theta = 56.1° \quad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point O, no moment is produced about this point.



(a)

4-86. Two couples act on the cantilever beam. If $F = 6$ kN, determine the resultant couple moment.



a) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by the 6- kN and 5 - kN couples, respectively, are given by

$$\circlearrowleft +(M_c)_1 = 6\sin 30°(3) - 6\cos 30°(0.5 + 0.5) = 3.804 \text{ kN} \cdot \text{m}$$

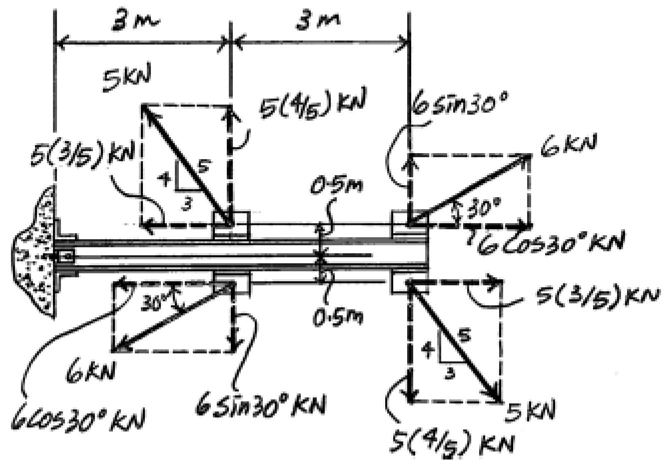
$$\circlearrowleft +(M_c)_2 = 5\left(\frac{3}{5}\right)(0.5 + 0.5) - 5\left(\frac{4}{5}\right)(3) = -9 \text{ kN} \cdot \text{m}$$

Thus, the resultant couple moment can be determined from

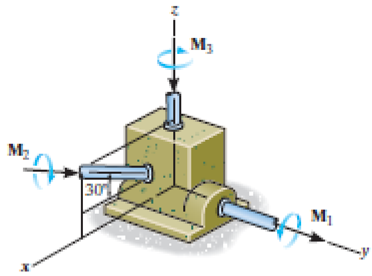
$$(M_c)_R = (M_c)_1 + (M_c)_2$$
$$= 3.804 - 9 = -5.196 \text{ kN} \cdot \text{m} = 5.20 \text{ kN} \cdot \text{m (clockwise)}$$ **Ans.**

b) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, and summing the moments of these force components about point A, we can write

$$\circlearrowleft +(M_c)_R = \Sigma M_A; \quad (M_c)_R = 5\left(\frac{3}{5}\right)(0.5) + 5\left(\frac{4}{5}\right)(3) - 6\cos 30°(0.5) - 6\sin 30°(3)$$
$$+ 6\sin 30°(6) - 6\cos 30°(0.5) + 5\left(\frac{3}{5}\right)(0.5) - 5\left(\frac{4}{5}\right)(6)$$
$$= -5.196 \text{ kN} \cdot \text{m} = 5.20 \text{ kN} \cdot \text{m (clockwise)}$$ **Ans.**



***4–92.** Determine the required magnitude of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is $\mathbf{M}_R = \{-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}\}$ N · m.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M}_1 = M_1\mathbf{j}$

$\mathbf{M}_2 = M_2(-\cos 30°\mathbf{i} - \sin 30°\mathbf{k}) = -0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}$

$\mathbf{M}_3 = -M_3\mathbf{k}$

The resultant couple moment is given by

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$ $\quad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

$(-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}) = M_1\mathbf{j} + (-0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}) + (-M_3\mathbf{k})$

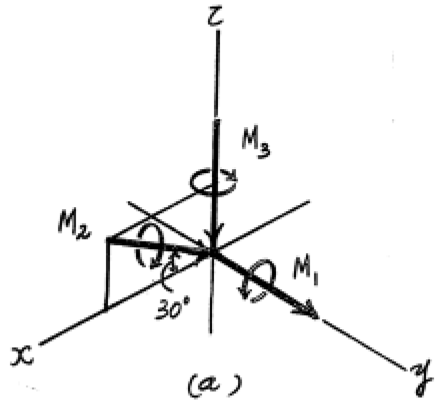
$-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k} = -0.8660M_2\mathbf{i} + M_1\mathbf{j} - (0.5M_2 + M_3)\mathbf{k}$

Equating the **i**, **j**, and **k** components yields

$-300 = -0.8660M_2$ $\quad M_2 = 346.41$ N · m = 346 N · m **Ans.**

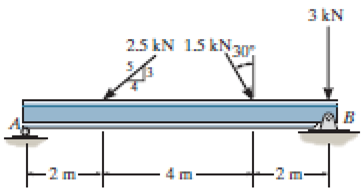
$M_1 = 450$ N · m **Ans.**

$600 = -0.5(346.41) + M_3$ $\quad M_3 = 427$ N · m **Ans.**



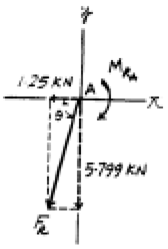
(a)

•4–105. Replace the force system acting on the beam by an equivalent force and couple moment at point *A*.



$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 1.5\sin 30^\circ - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25\ \text{kN} = 1.25\ \text{kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5\cos 30^\circ - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799\ \text{kN} = 5.799\ \text{kN} \downarrow$$



Thus,

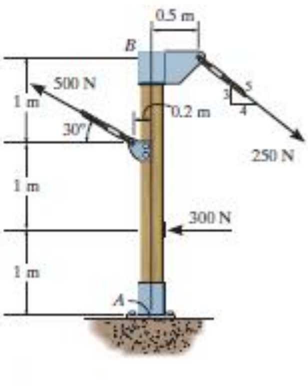
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93\ \text{kN} \qquad \textbf{Ans}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8^\circ \qquad \textbf{Ans}$$

$$\curvearrowleft + M_{R_A} = \Sigma M_A; \quad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5\cos 30^\circ(6) - 3(8)$$
$$= -34.8\ \text{kN}\cdot\text{m} = 34.8\ \text{kN}\cdot\text{m}\ \ (\textit{Clockwise}) \qquad \textbf{Ans}$$

•4–109. Replace the force system acting on the post by a resultant force and couple moment at point A.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y;\quad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N}$$

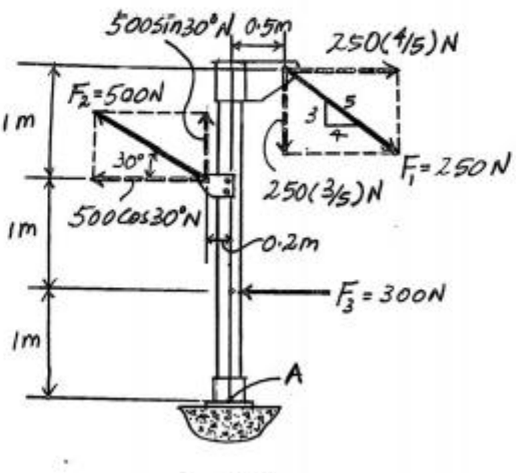
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ \quad \nwarrow$$ **Ans.**

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a, and summing the moments of the force components algebraically about point A,

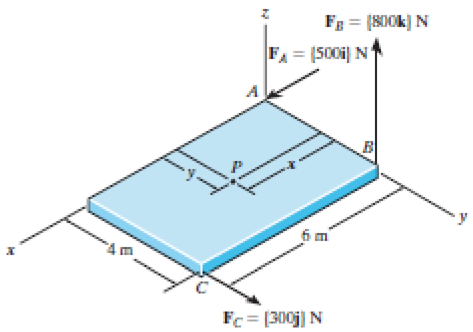
$$\circlearrowleft +(M_R)_A = \Sigma M_A;\quad (M_R)_A = 500\cos 30^\circ(2) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$= 441.02\text{ N}\cdot\text{m} = 441\text{ N}\cdot\text{m (counterclockwise)}$$ **Ans.**



(a)

•**4–141.** Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}$ N

$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990$ N **Ans**

$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$

$M_{R_{x'}} = \Sigma M_{x'};\qquad M_{R_{x'}} = 800(4-y)$

$M_{R_{y'}} = \Sigma M_{y'};\qquad M_{R_{y'}} = 800x$

$M_{R_{z'}} = \Sigma M_{z'};\qquad M_{R_{z'}} = 500y + 300(6-x)$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

$M_R(0.5051) = 800(4-y)$

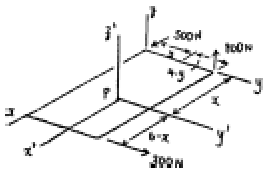
$M_R(0.3030) = 800x$

$M_R(0.8081) = 500y + 300(6-x)$

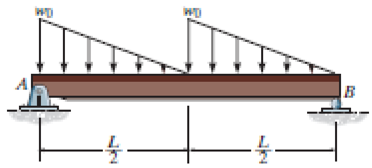
$M_R = 3.07$ kN·m **Ans**

$x = 1.16$ m **Ans**

$y = 2.06$ m **Ans**



•**4–145.** Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into two parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also shown in Fig. *a*.

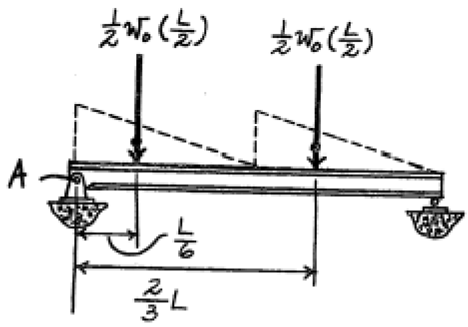
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$+\downarrow F_R = \Sigma F;\qquad F_R = \frac{1}{2}w_0\left(\frac{L}{2}\right)+\frac{1}{2}w_0\left(\frac{L}{2}\right)=\frac{1}{2}w_0L\downarrow$ **Ans.**

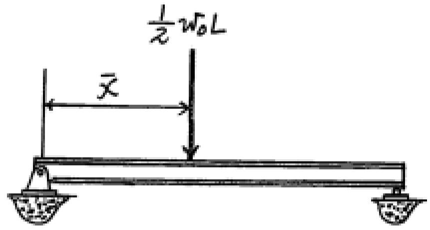
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point *A*,

$\circlearrowleft+(M_R)_A = \Sigma M_A;\quad -\frac{1}{2}w_0L(\bar{x}) = -\frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{6}\right)-\frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{2}{3}L\right)$

$\bar{x}=\frac{5}{12}L$ **Ans.**



(a)



(b)