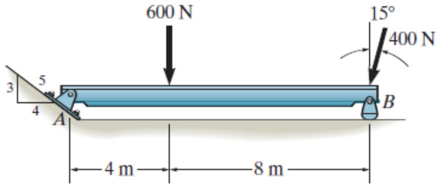
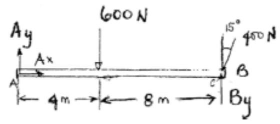
CHAPTER V (SELECTED PROBLEMS)

5–11.

Determine the magnitude of the reactions on the beam at A and B. Neglect the thickness of the beam.



SOLUTION



$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(12) - (400\cos 15°)(12) - 600(4) = 0$

$B_y = 586.37 = 586 \text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 400 \sin 15° = 0$

$A_x = 103.528 \text{ N}$

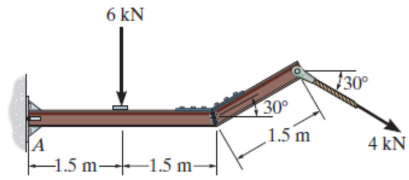
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 600 - 400 \cos 15° + 586.37 = 0$

$A_y = 400 \text{ N}$

$F_A = \sqrt{(103.528)^2 + (400)^2} = 413 \text{ N}$ **Ans.**

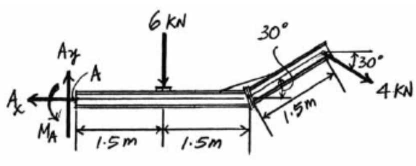
5–12.

Determine the components of the support reactions at the fixed support A on the cantilevered beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.



(a)

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 4 \cos 30° - A_x = 0$

$A_x = 3.46 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 6 - 4 \sin 30° = 0$

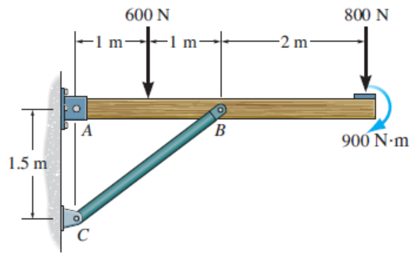
$A_y = 8 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4\cos 30°(1.5 \sin 30°) - 4 \sin 30°(3 + 1.5\cos 30°) = 0$

$M_A = 20.2 \text{ kN}\cdot\text{m}$ **Ans.**

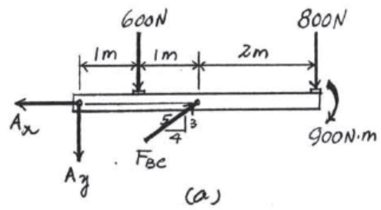
5–14.

The overhanging beam is supported by a pin at A and the two-force strut BC. Determine the horizontal and vertical components of reaction at A and the reaction at B on the beam.



SOLUTION

Equations of Equilibrium: Since line BC is a two-force member, it will exert a force $\mathbf{F}_{BC}$ directed along its axis on the beam as shown on the free-body diagram, Fig. *a*. From the free-body diagram, F_{BC} can be obtained by writing the moment equation of equilibrium about point A.



$\circlearrowleft + \Sigma M_A = 0;$ $\qquad F_{BC}\left(\frac{3}{5}\right)(2) - 600(1) - 800(4) - 900 = 0$

$F_{BC} = 3916.67 \text{ N} = 3.92 \text{ kN}$ **Ans.**

Using this result and writing the force equations of equilibrium along the x and y axes, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad 3916.67\left(\frac{4}{5}\right) - A_x = 0$

$A_x = 3133.33 \text{ N} = 3.13 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad -A_y - 600 - 800 + 3916.67\left(\frac{3}{5}\right) = 0$

$A_y = 950 \text{ N}$ **Ans.**

Determine the reactions on the bent rod which is supported by a smooth surface at *B* and by a collar at *A*, which is fixed to the rod and is free to slide over the fixed inclined rod.

Given:

$F := 100N$

$M := 20N \cdot m$

$a := 0.3m$

$b := 0.3m$

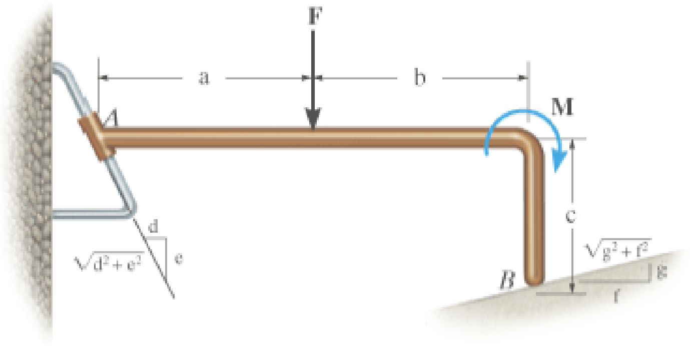
$c := 0.2m$

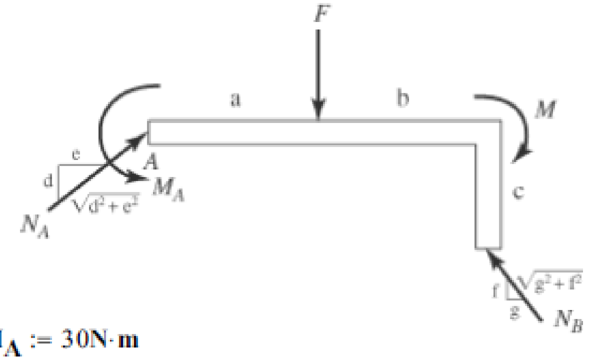
$d := 3$

$e := 4$

$f := 12$

$g := 5$





Solution:

Initial Guesses:

$N_A := 20N \qquad N_B := 10N \qquad M_A := 30N \cdot m$

Given

$$\Sigma M_A = 0; \quad M_A - F \cdot a - M + N_B \cdot \frac{f}{\sqrt{f^2+g^2}} \cdot (a+b) - N_B \cdot \frac{g}{\sqrt{f^2+g^2}} \cdot c = 0$$

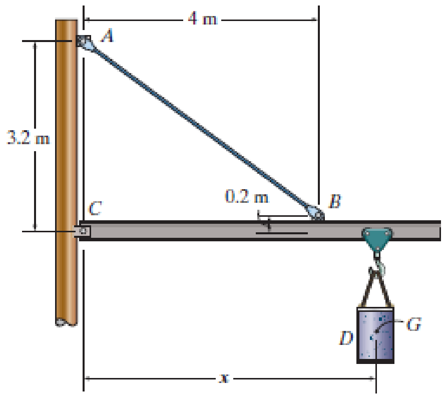
$$\Sigma F_x = 0; \quad N_A \cdot \frac{e}{\sqrt{e^2+d^2}} - N_B \cdot \frac{g}{\sqrt{f^2+g^2}} = 0$$

$$\Sigma F_y = 0; \quad N_A \cdot \frac{d}{\sqrt{e^2+d^2}} + N_B \cdot \frac{f}{\sqrt{f^2+g^2}} - F = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ M_A \end{pmatrix} := \text{Find}(N_A, N_B, M_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 39.7 \\ 82.5 \end{pmatrix} N \qquad M_A = 10.6\, N \cdot m$$

Ans.

5–31. The jib crane is supported by a pin at C and rod AB. If the load has a mass of 2 Mg with its center of mass located at G, determine the horizontal and vertical components of reaction at the pin C and the force developed in rod AB on the crane when $x = 5$ m.



Equations of Equilibrium: Realizing that rod AB is a two - force member, it will exert a force $\mathbf{F}_{AB}$ directed along its axis on the beam, as shown on the free - body diagram in Fig. *a*. From the free - body diagram, F_{AB} can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\left(\frac{3}{5}\right)(4) + F_{AB}\left(\frac{4}{5}\right)(0.2) - 2000(9.81)(5) = 0$$

$$F_{AB} = 38\,320.31 \text{ N} = 38.3 \text{ kN} \qquad \textbf{Ans.}$$

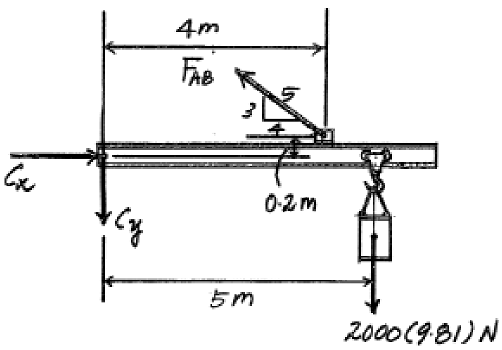
Using the above result and writing the force equations of equilibrium along the x and y axes.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 38\,320.31\left(\frac{4}{5}\right) = 0$$

$$C_x = 30\,656.25 \text{ N} = 30.7 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 38\,320.31\left(\frac{3}{5}\right) - 2000(9.81) - C_y = 0$$

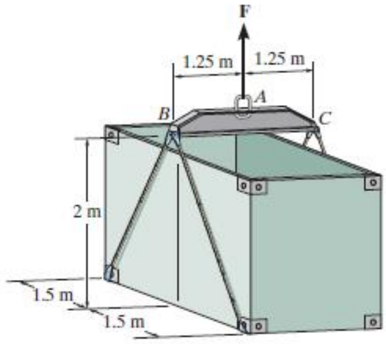
$$C_y = 3372.19 \text{ N} = 3.37 \text{ kN} \qquad \textbf{Ans.}$$



(a)

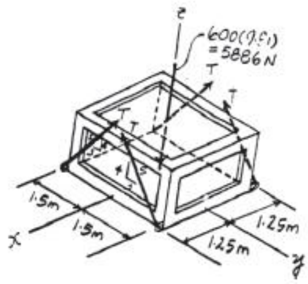
5–62.

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at A.



SOLUTION

Equations of Equilibrium: Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the x and y axes and force equilibrium along y axis.



$\Sigma F_z = 0;$ $$4T\left(\frac{4}{5}\right) - 5886 = 0$$

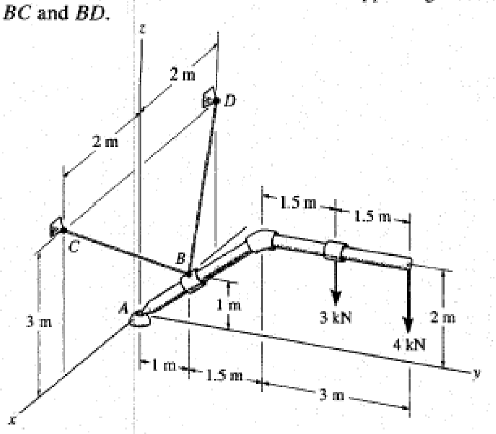
$$T = 1839.375 \text{ N} = 1.84 \text{ kN}$$ **Ans.**

The force **F** applied to the sling A must support the weight of the load and strongback beam. Hence

$\Sigma F_z = 0;$ $$F - 600(9.81) - 30(9.81) = 0$$

$$F = 6180.3 \text{ N} = 6.18 \text{ kN}$$ **Ans.**

5-66. The pipe assembly supports the vertical loads shown. Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables BC and BD.

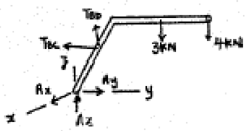


$$\mathbf{T}_{BD} = T_{BD}\left(\frac{-2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$\Sigma M_x = 0;$ $$-3(4) - 4(5.5) + \frac{2}{3}T_{BD}(1) + \frac{2}{3}T_{BC}(1) + \frac{1}{3}T_{BD}(1) + \frac{1}{3}T_{BC}(1) = 0$$

$$T_{BD} + T_{BC} = 34$$



$\Sigma M_y = 0;$ $$\frac{2}{3}T_{BC}(1) - \frac{2}{3}T_{BD} = 0$$

$$T_{BC} = T_{BD}$$

$$T_{BC} = T_{BD} = 17 \text{ kN}$$ **Ans.**

$\Sigma F_y = 0;$ $$A_y - 17(\tfrac{1}{3}) - 17(\tfrac{1}{3}) = 0$$

$$A_y = 11.3 \text{ kN}$$ **Ans.**

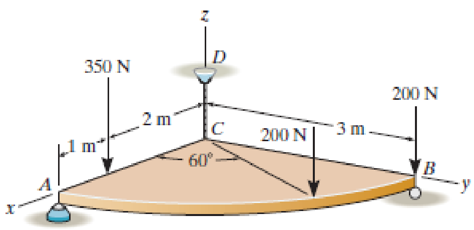
$\Sigma F_x = 0;$ $$A_x = 0$$ **Ans.**

$\Sigma F_z = 0;$ $$A_z + 17(\tfrac{2}{3}) + 17(\tfrac{2}{3}) - 3 - 4 = 0$$

$$A_z = -15.7 \text{ kN}$$ **Ans.**

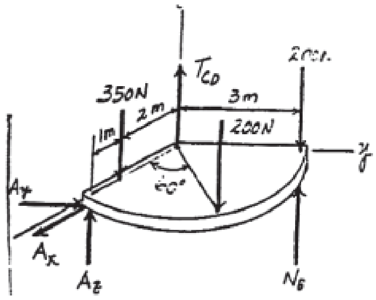
5–68.

Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



SOLUTION

Equations of Equilibrium: The normal reactions N_B and A_z can be obtained directly by summing moments about the x and y axes, respectively.



$\Sigma M_x = 0;\quad N_B(3) - 200(3) - 200(3 \sin 60°) = 0$

$$N_B = 373.21\text{ N} = 373\text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_y = 0;\quad 350(2) + 200(3 \cos 60°) - A_z(3) = 0$

$$A_z = 333.33\text{ N} = 333\text{ N} \qquad \textbf{Ans.}$$

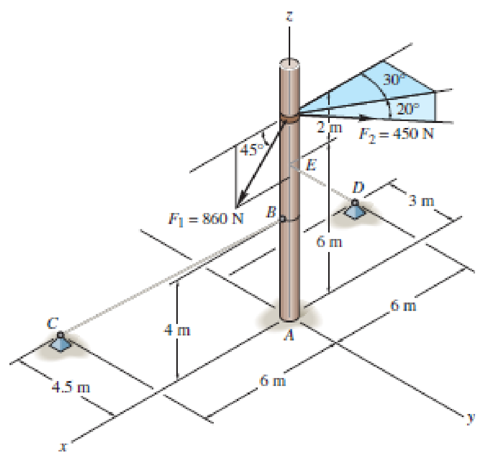
$\Sigma F_z = 0;\quad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

$$T_{CD} = 43.5\text{ N} \qquad \textbf{Ans.}$$

$\Sigma F_x = 0;\quad A_x = 0 \qquad$ **Ans.**

$\Sigma F_y = 0;\quad A_y = 0 \qquad$ **Ans.**

The pole is subjected to the two forces shown. Determine the components of reaction of A assuming it to be a ball-and-socket joint. Also, compute the tension in each of the guy wires, BC and ED.



SOLUTION

Force Vector and Position Vectors:

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{F}_1 = 860\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\ \text{N} = \{608.11\mathbf{i} - 608.11\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = 450\{-\cos 20° \cos 30°\mathbf{i} + \cos 20° \sin 30°\mathbf{k} - \sin 20°\mathbf{k}\}\ \text{N}$$

$$= \{-366.21\mathbf{i} + 211.43\mathbf{j} - 153.91\mathbf{k}\}\ \text{N}$$

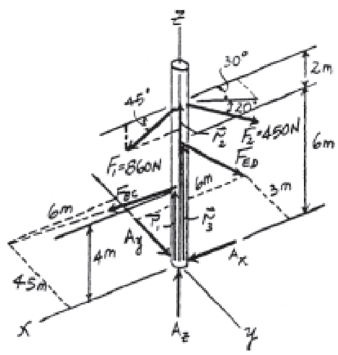
$$\mathbf{F}_{ED} = F_{ED}\left[\frac{(-6-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-6-0)^2 + (-3-0)^2 + (0-6)^2}}\right]$$

$$= -\frac{2}{3}F_{ED}\mathbf{i} - \frac{1}{3}F_{ED}\mathbf{j} - \frac{2}{3}F_{ED}\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC}\left[\frac{(6-0)\mathbf{i} + (-4.5-0)\mathbf{j} + (0-4)\mathbf{k}}{\sqrt{(6-0)^2 + (-4.5-0)^2 + (0-4)^2}}\right]$$

$$= \frac{12}{17}F_{BC}\mathbf{i} - \frac{9}{17}F_{BC}\mathbf{j} - \frac{8}{17}F_{BC}\mathbf{k}$$

$$\mathbf{r}_1 = \{4\mathbf{k}\}\ \text{m} \qquad \mathbf{r}_2 = \{8\mathbf{k}\}\ \text{m} \qquad \mathbf{r}_3 = \{6\mathbf{k}\}\ \text{m}$$



Equations of Equilibrium: Force equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_A + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_{ED} + \mathbf{F}_{BC} = \mathbf{0}$$

$$\left(A_x + 608.11 - 366.21 - \frac{2}{3}F_{ED} + \frac{12}{17}F_{BC}\right)\mathbf{i}$$

$$+ \left(A_y + 211.43 - \frac{1}{3}F_{ED} - \frac{9}{17}F_{BC}\right)\mathbf{j}$$

$$+ \left(A_z - 608.11 - 153.91 - \frac{2}{3}F_{ED} - \frac{8}{17}F_{BC}\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

$$\Sigma F_x = 0; \qquad A_x + 608.11 - 366.21 - \frac{2}{3}F_{ED} + \frac{12}{17}F_{BC} = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad A_y + 211.43 - \frac{1}{3}F_{ED} - \frac{9}{17}F_{BC} = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \qquad A_z - 608.11 - 153.91 - \frac{2}{3}F_{ED} - \frac{8}{17}F_{BC} = 0 \qquad \textbf{(3)}$$

5–72. (continued)

Moment equilibrium requires

$$\Sigma \mathbf{M}_A = \mathbf{0}; \qquad \mathbf{r}_1 \times \mathbf{F}_{BC} + \mathbf{r}_2 \times (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{r}_3 \times \mathbf{F}_{ED} = \mathbf{0}$$

$$4\mathbf{k} \times \left(\frac{12}{17}F_{BC}\mathbf{i} - \frac{9}{17}F_{BC}\mathbf{j} - \frac{8}{17}F_{BC}\mathbf{k}\right)$$

$$+ 8\mathbf{k} \times (241.90\mathbf{i} + 211.43\mathbf{j} - 762.02\mathbf{k})$$

$$+ 6\mathbf{k} \times \left(-\frac{2}{3}F_{ED}\mathbf{i} - \frac{1}{3}F_{ED}\mathbf{j} - \frac{2}{3}F_{ED}\mathbf{k}\right) = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

$$\Sigma M_x = 0; \qquad \frac{36}{17}F_{BC} + 2F_{ED} - 1691.45 = 0 \qquad \textbf{(4)}$$

$$\Sigma M_y = 0; \qquad \frac{48}{17}F_{BC} - 4F_{ED} + 1935.22 = 0 \qquad \textbf{(5)}$$

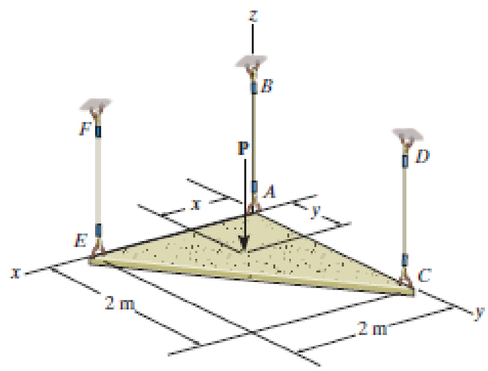
Solving Eqs. (4) and (5) yields

$F_{BC} = 205.09\text{ N} = 205\text{ N} \qquad F_{ED} = 628.57\text{ N} = 629\text{ N}$ **Ans.**

Substituting the results into Eqs. (1), (2) and (3) yields

$A_x = 32.4\text{ N} \qquad A_y = 107\text{ N} \qquad A_z = 1277.58\text{ N} = 1.28\text{ kN}$ **Ans.**

5–73. If $P = 6$ kN, $x = 0.75$ m and $y = 1$ m, determine the tension developed in cables AB, CD, and EF. Neglect the weight of the plate.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, T_{CD} and T_{EF} can be obtained by writing the moment equation of equilibrium about the *x* and *y* axes, respectively.

$\Sigma M_x = 0;\quad T_{CD}(2) - 6(1) = 0$

$T_{CD} = 3\text{ kN}$ **Ans.**

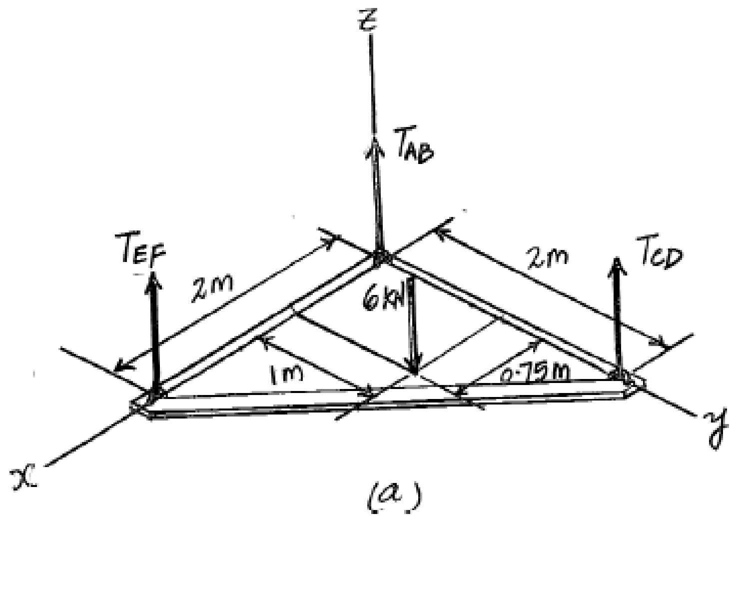
$\Sigma M_y = 0;\quad T_{EF}(2) - 6(0.75) = 0$

$T_{EF} = 2.25\text{ kN}$ **Ans.**

Using the above results and writing the force equation of equilibrium along the *z* axis,

$\Sigma F_z = 0;\quad T_{AB} + 3 + 2.25 - 6 = 0$

$T_{AB} = 0.75\text{ kN}$ **Ans.**



(a)